# Monitoring edge-geodetic sets of chordal graphs

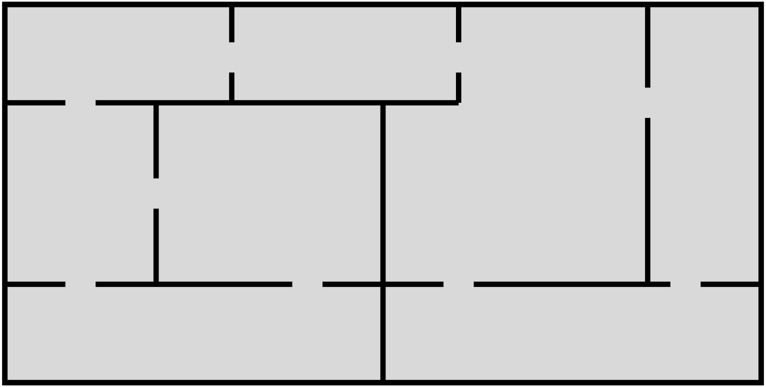
Clara Marcille[1] Nacim Oijid[2]

[1]LIP, ENS de Lyon

[2]Umeå University

IRIF Graph Seminar, 27 January 2026

# Detecting Events in Graphs



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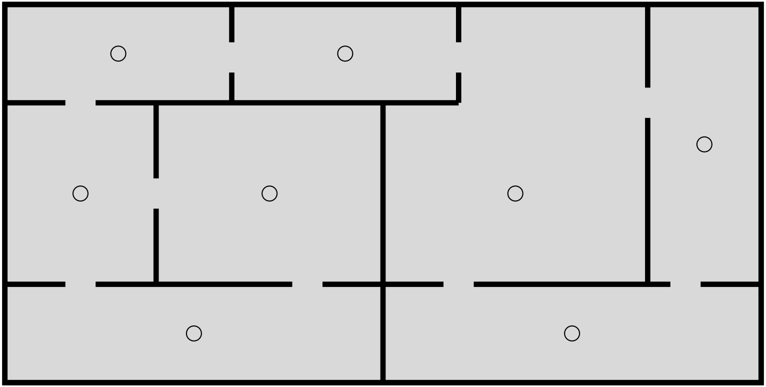

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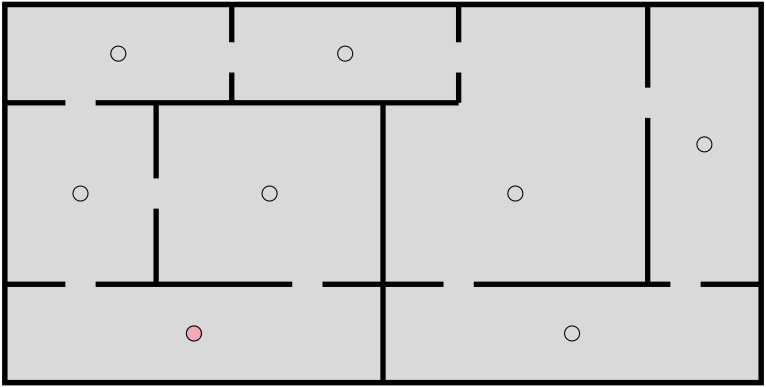

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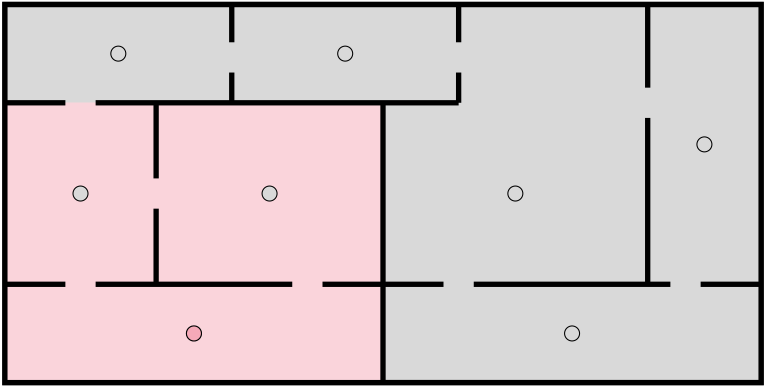

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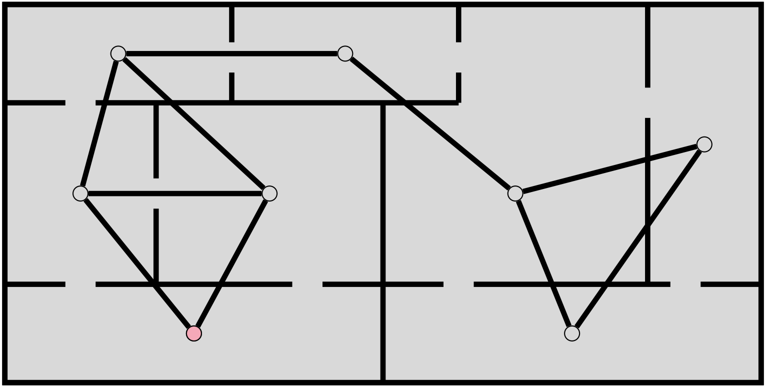

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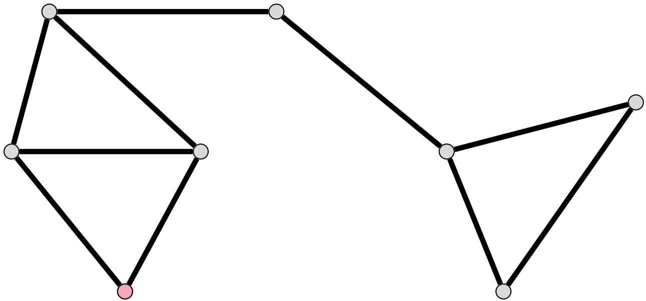

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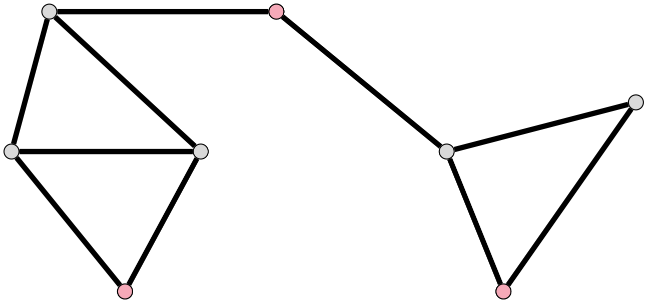

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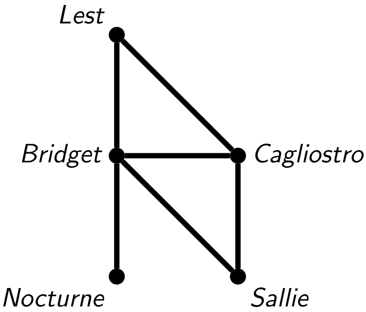

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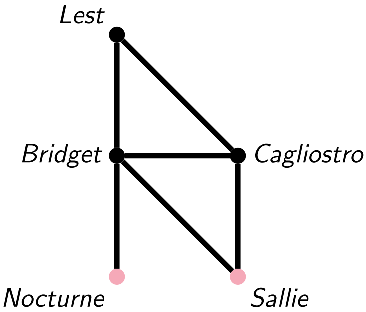

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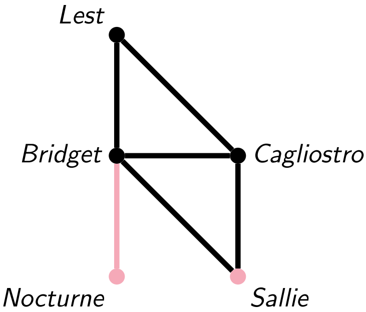

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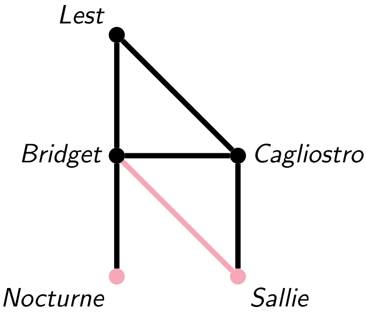

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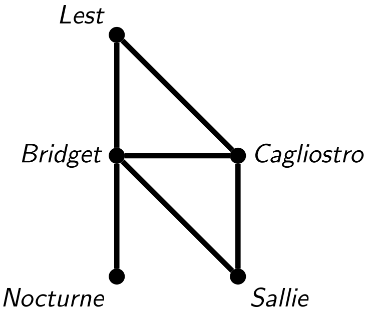

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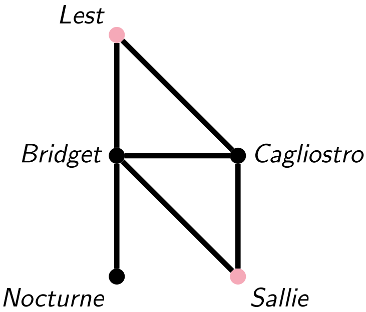

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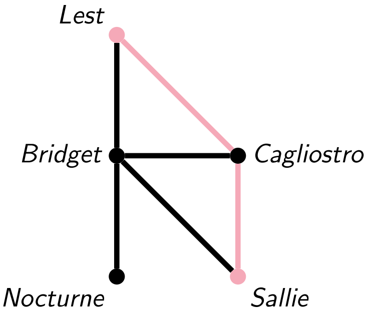

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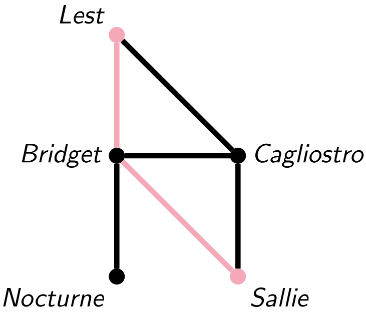

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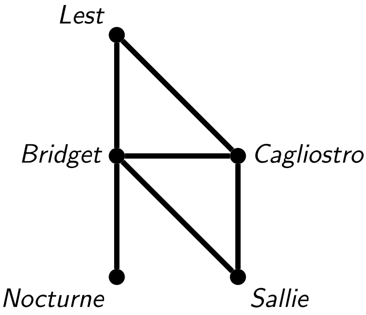

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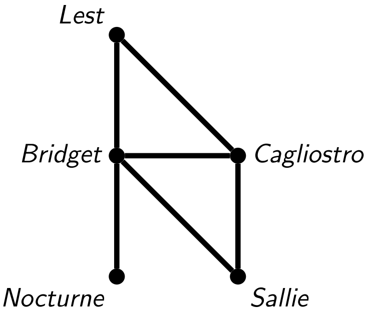


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- every step takes the same time;
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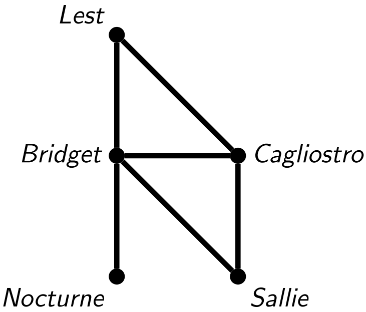


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This object simulates probes monitoring a network: if the value of the ping between two probes increases, then one can know a failure happened.

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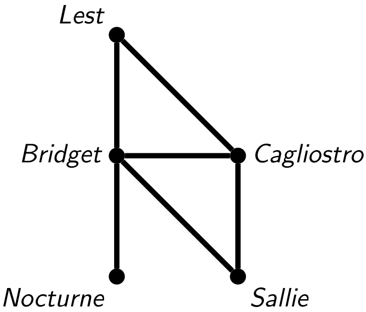


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## Goal

We want to minimise the number of probes.

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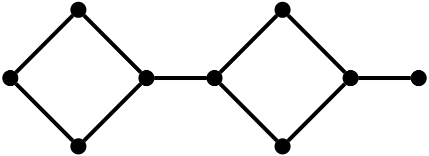

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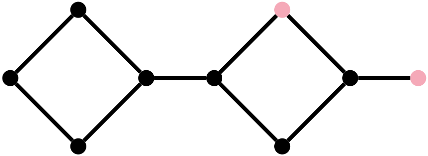

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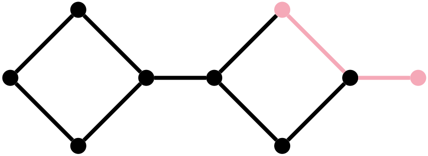

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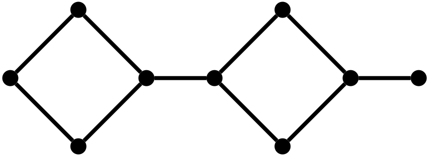

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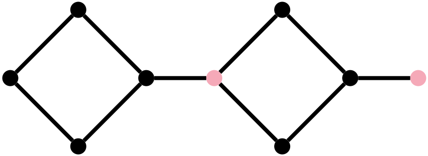

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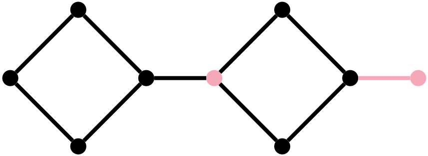

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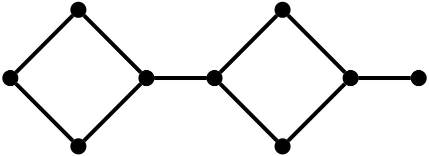

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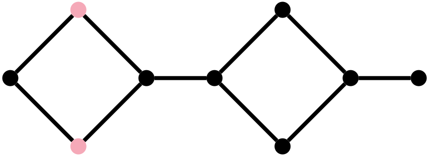

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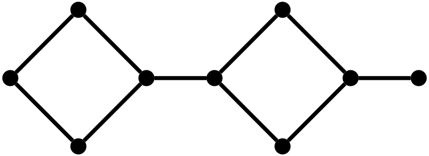

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A set $M$ of vertices *monitors* an edge $e$ if $e$ lies on all shortest paths between two vertices of $M$.

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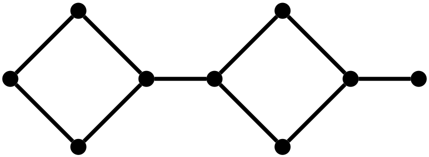

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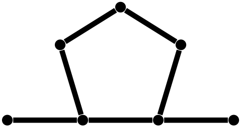

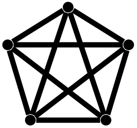

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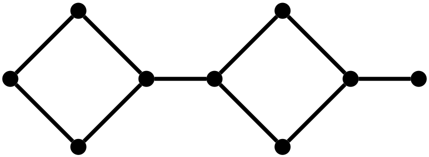

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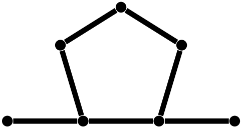

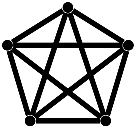

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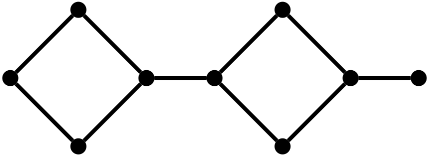

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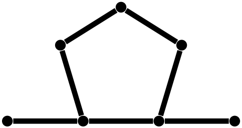

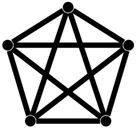

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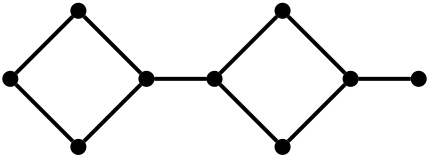

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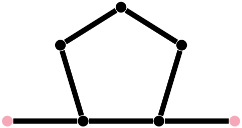

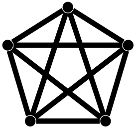

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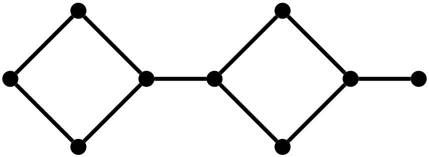

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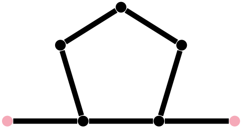

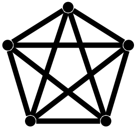

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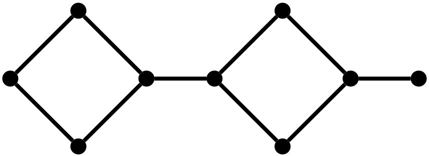

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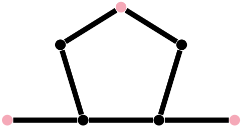

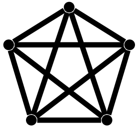

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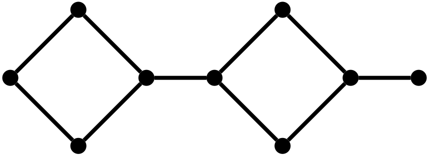

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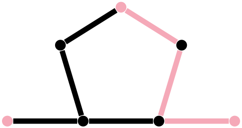

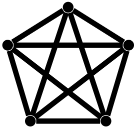

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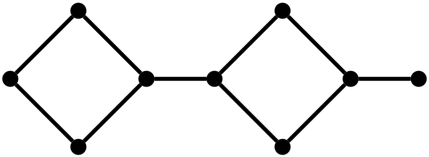

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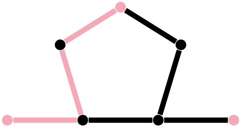

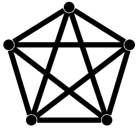

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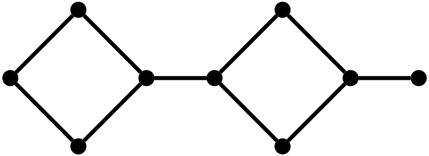

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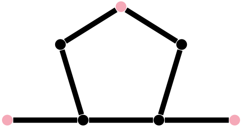

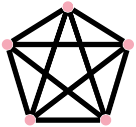

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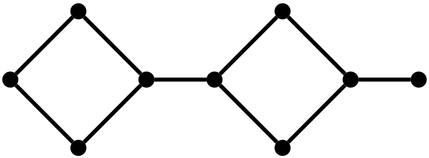

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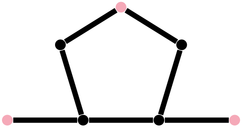

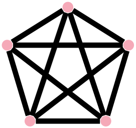

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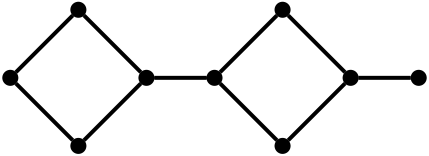

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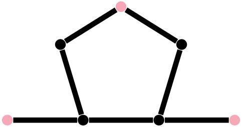

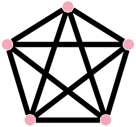

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Parameterised Complexity

For a graph $G$, there is a polynomial-time algorithm to:

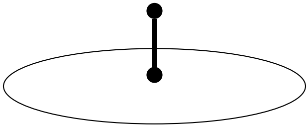
- find a MEG-set of $G$ of size $meg(G) \cdot \sqrt{n \ln m}$, but
- not of size $c \cdot meg(G)$ for any $c > 1$.

Approximation Algorithms

# Mandatory Vertices

**Definition**

A vertex of $G$ is *mandatory* if it belongs to all MEG-sets of $G$.



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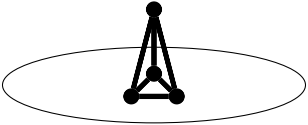

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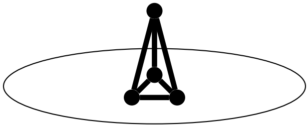

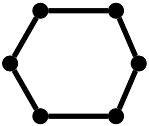

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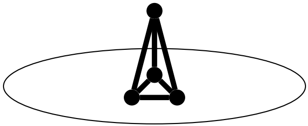

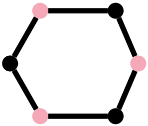

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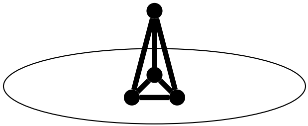

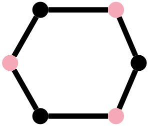

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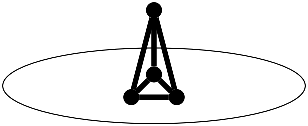

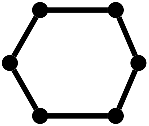

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We can find the set of mandatory vertices of a graph in polynomial time.

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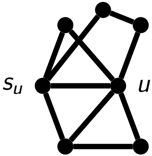


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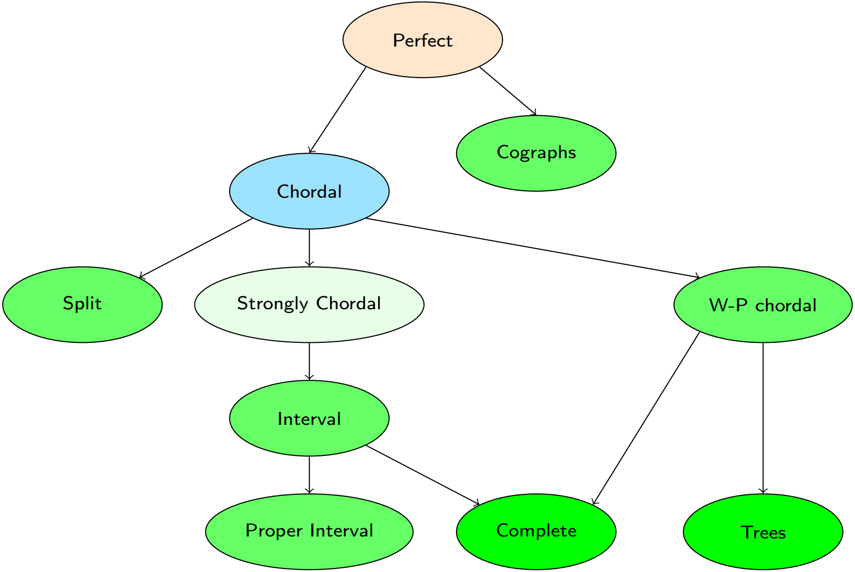
# MEG-minimal Graph Classes



# Idea of the Interval Graphs Result

**Lemma [FMMSST24]**

In an interval graph, a vertex is mandatory if and only if its neighbourhood has diameter at most 4.

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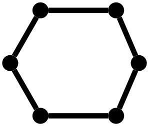

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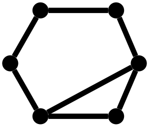

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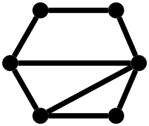

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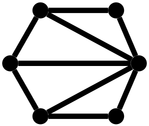

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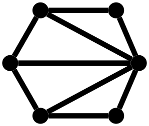

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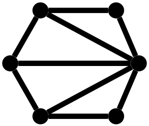

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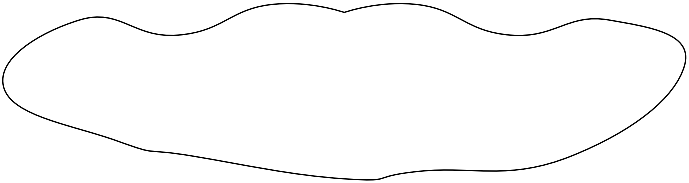

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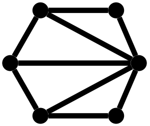

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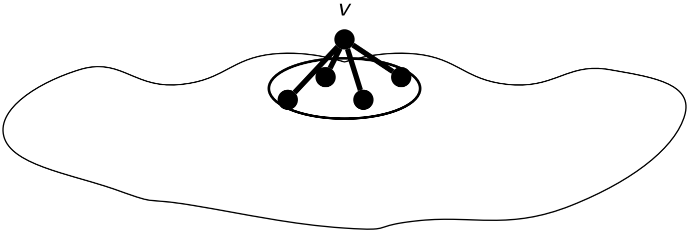

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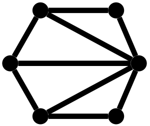

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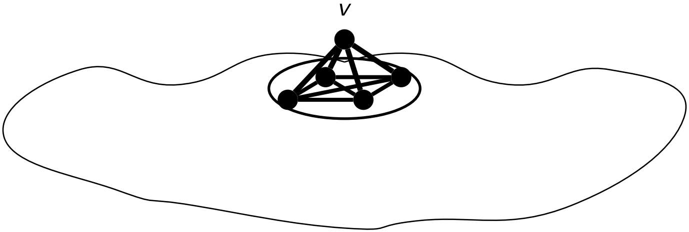

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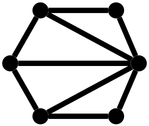

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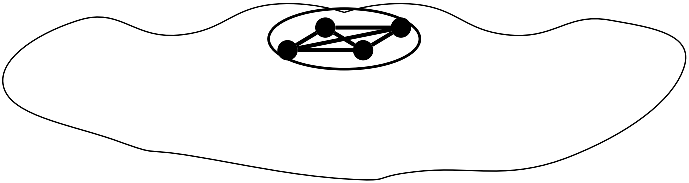

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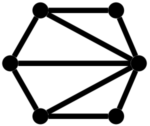

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**Theorem [FNRS23]**

Simplicial vertices are mandatory.

# Overview of the Proof

## Idea of the proof

- Consider a minimal counterexample $G$ and remove $v$ the first vertex of an elimination scheme of $G$.
- Look at $W$ the vertices mandatory in $G - v$ but not in $G$, and prove $W \subset N(v)$.
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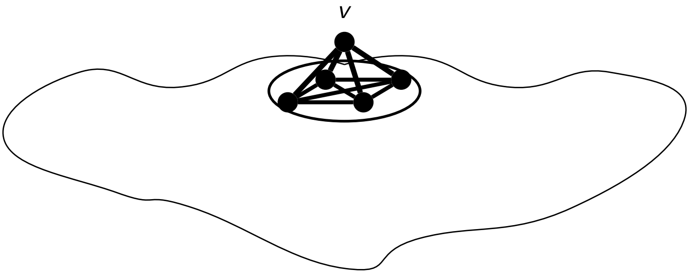

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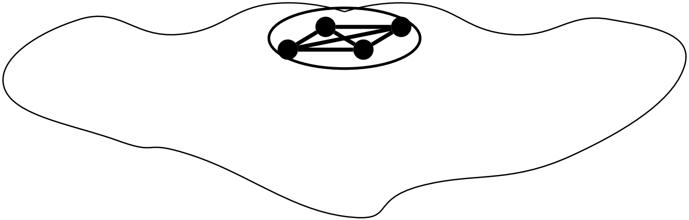

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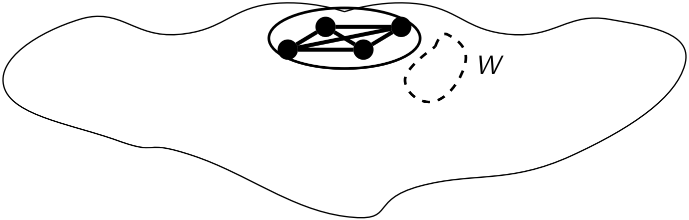

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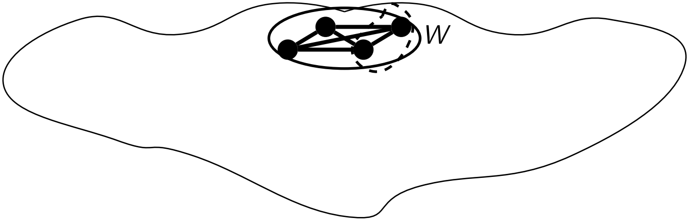

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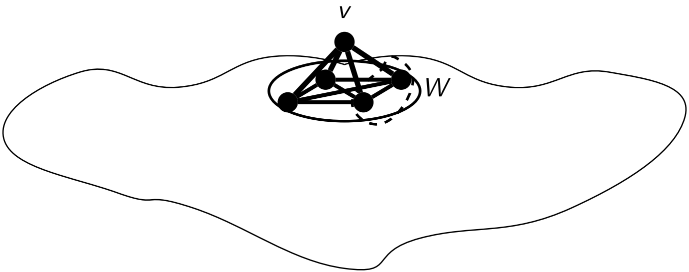

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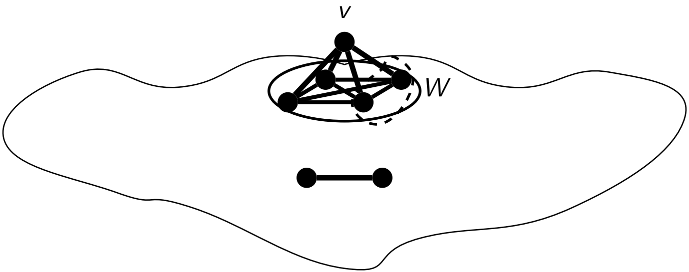

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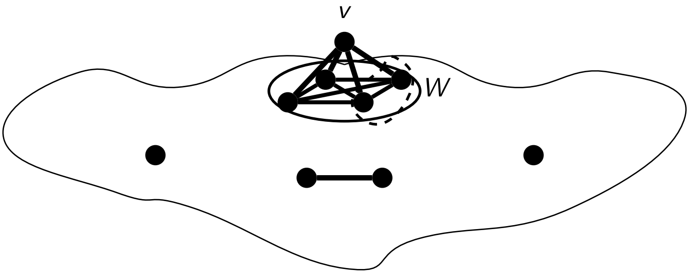

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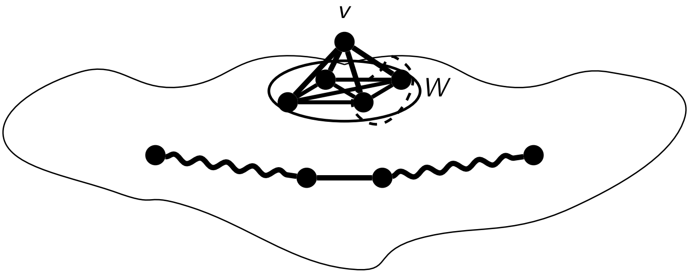

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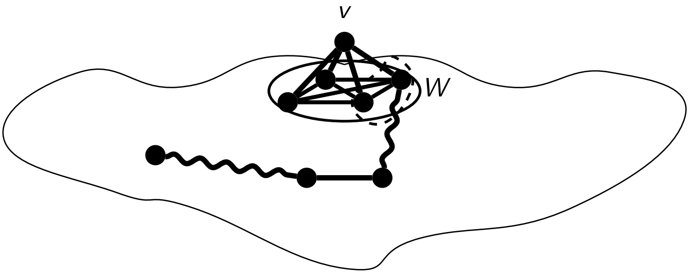

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Theorem [FMMSST24]

A vertex $u$ is *mandatory* if there exists $s_u$ such that for any $x \in N(u)$ such that $s_u ux$ is an induced 2-path, there exists $w \in V(G)$ such that $s_u uxw$ is a cycle.

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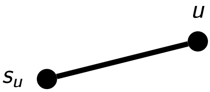

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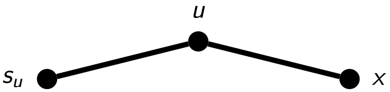

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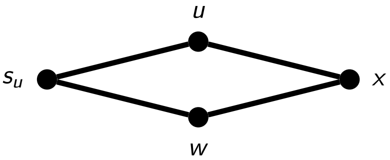

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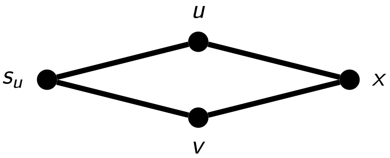

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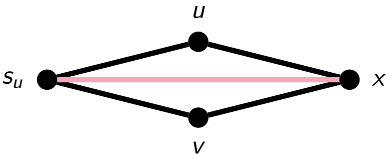

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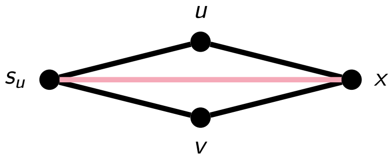


**Corollary**

We look at the edges monitored by a vertex of $W$.

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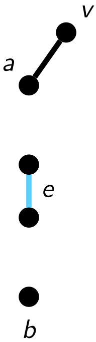

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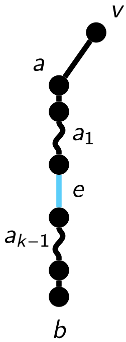

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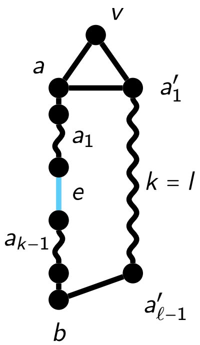

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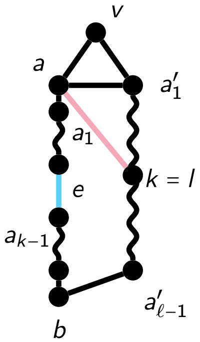

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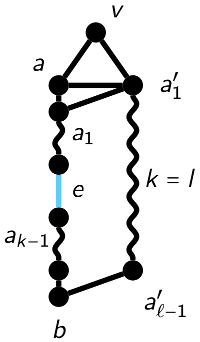

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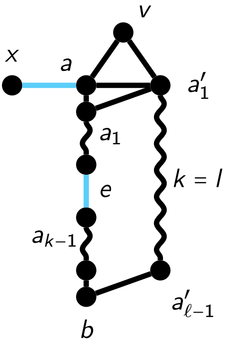


**Theorem [FMMSST24]**

A vertex $u$ is *mandatory* if there exists $s_u$ such that for any $x \in N(u)$ such that $s_u ux$ is an induced 2-path, there exists $w \in V(G)$ such that $s_u uxw$ is a cycle.

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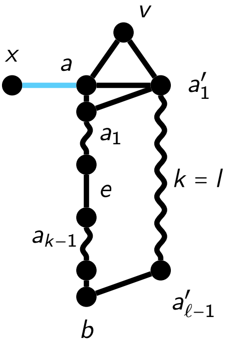


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A vertex $a$ is *mandatory* if there exists $a_1$ such that for any $x \in N(u)$ such that $a_1ax$ is an induced 2-path, there exists $z \in V(G)$ such that $a_1axz$ is a cycle.

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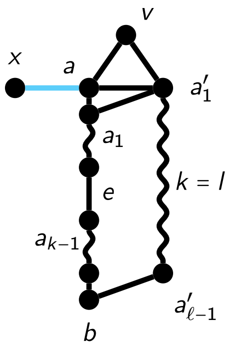


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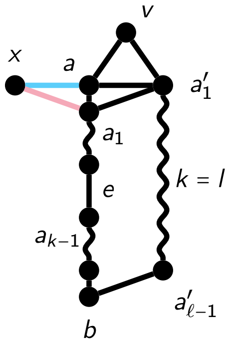


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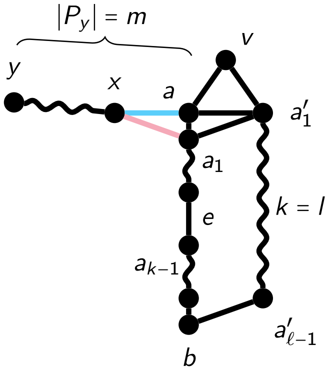


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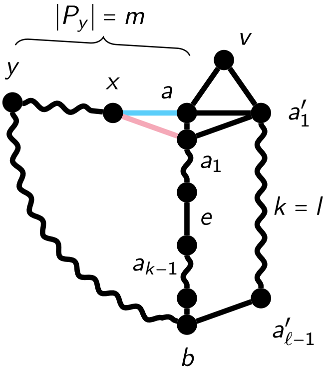


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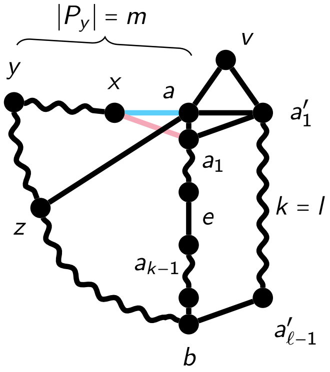


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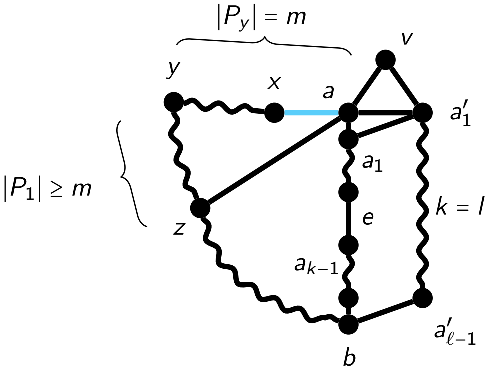


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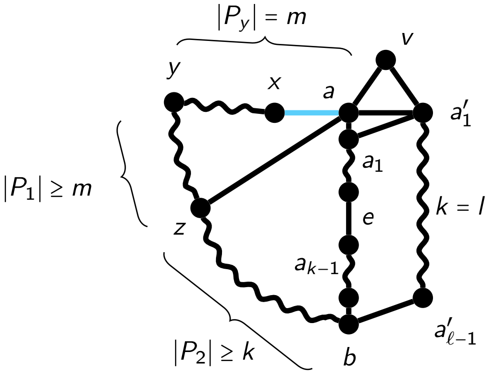


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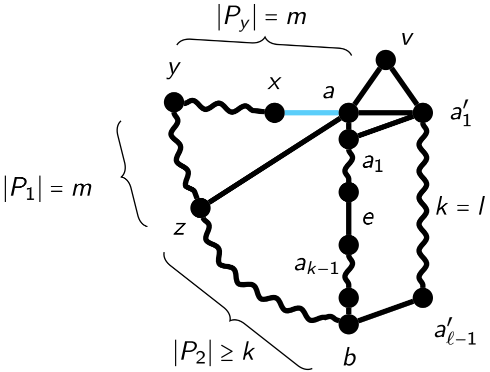


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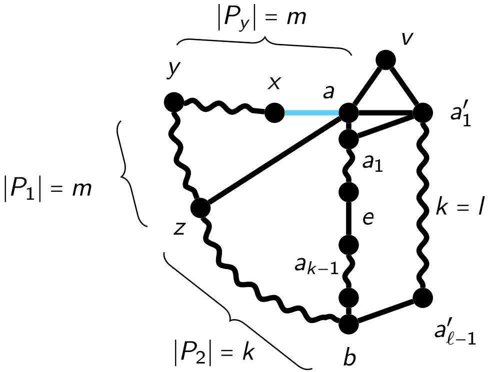


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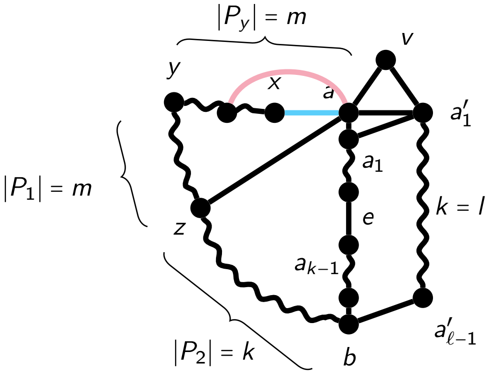


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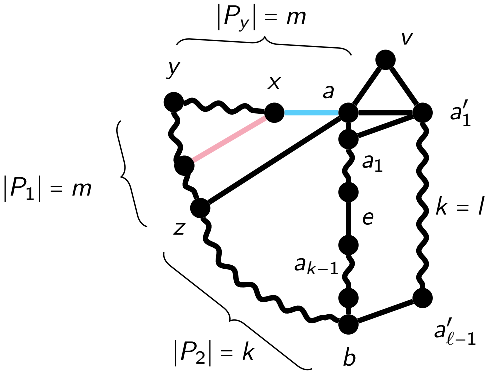


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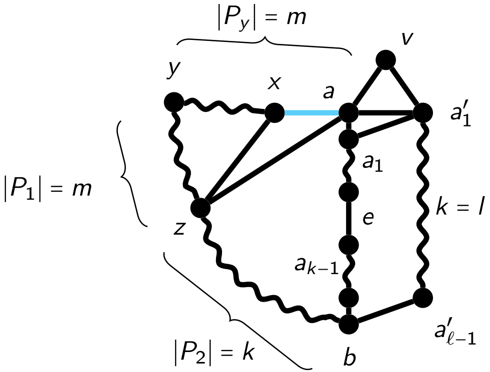


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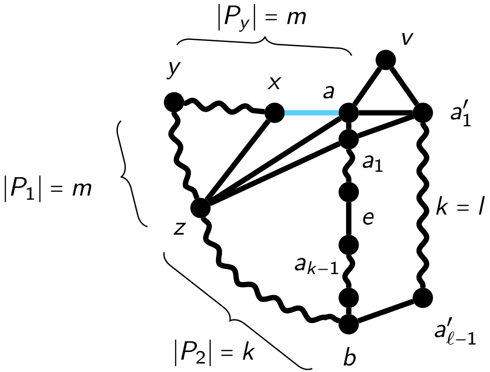


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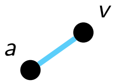

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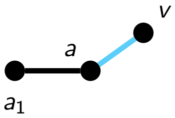

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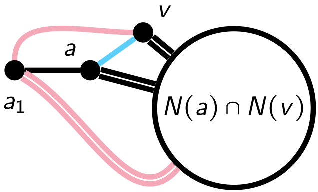

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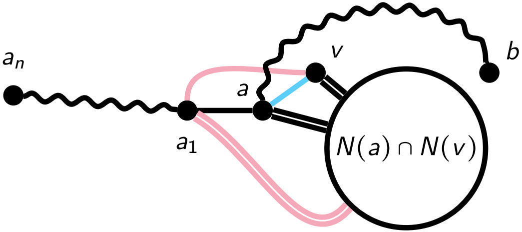

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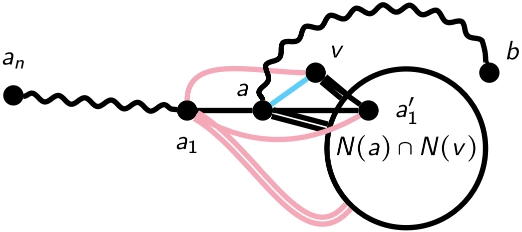

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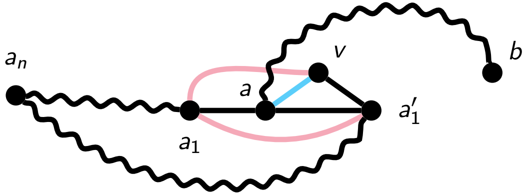

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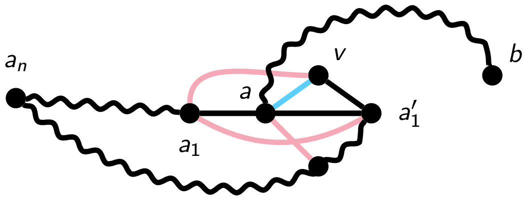

# Conclusion and Future Works

In this work:

- we compute MEG-sets of chordal graphs in polynomial time;
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- we compute MEG-sets of chordal graphs in polynomial time;
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What this result opens:

- Finding more MEG-minimal classes of graphs (all the way up to Perfect);
- investigate polynomial algorithms for non-MEG-minimal classes;
- new intuitions for parameterised algorithms.